Single-Transverse Spin Asymmetries:
From DIS to Hadronic Collisions

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Abstract

We study single-spin asymmetries in semi-inclusive deep inelastic scattering with transversely polarized target. Based on the QCD factorization approach, we consider Sivers and Collins contributions to the asymmetries. We fit simple parameterizations for the Sivers and Collins functions to the recent HERMES data, and compare to results from COMPASS. Using the fitted parameterizations for the Sivers functions, we predict the single transverse spin asymmetries for various processes in pp collisions at RHIC, including the Drell-Yan process and angular correlations in di-jet and jet-plus-photon production. These asymmetries are found to be sizable at forward rapidities.

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I.   INTRODUCTION

Single-transverse spin asymmetries (SSA) in hadronic processes have a long history, starting from the 1970s and 1980s when surprisingly large SSAs were observed in $p_\uparrow p \to \pi X$ [1] and $pp \to \Lambda_\uparrow X$ [2] at forward rapidities of the produced hadron. They have again attracted much interest in recent years from both experimental and theoretical sides [3]. In particular, first measurements by the STAR, PHENIX, and BRAHMS collaborations at RHIC have now become available [4, 5, 6], which extend the SSA observations from the fixed-target energy range to the collider regime. Again, large asymmetries were found in $p_\uparrow p \to \pi X$ at forward rapidities of the produced pion. Meanwhile, experimental studies in Deep Inelastic Scattering (DIS) by the HERMES collaboration at DESY, by SMC at CERN, and by CLAS at the Jefferson Laboratory also show remarkably large SSAs in semi-inclusive hadron production, $\gamma^* p_\uparrow \to \pi X$ [7, 8, 9, 10, 11]. Data from COMPASS for scattering off deuterons have been published as well [12], which show no large asymmetry. On the theoretical side, there are several approaches to understanding SSAs within Quantum Chromodynamics (QCD) [3, 13, 14]. Recent interest focuses on the role of partonic transverse momentum in creating the observed asymmetries. Transverse-momentum-dependent (TMD) parton distributions and fragmentation functions, and their relevance for semi-inclusive DIS (SIDIS), the Drell-Yan process, and single-inclusive hadron production at hadron colliders have been investigated in [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27]. Compared to the normal integrated distributions, the TMD distributions provide much more information; for example, some of them contain information on orbital angular momenta of partons in the nucleon and have also been linked to spatial distributions of partons [28, 29].

The Sivers function [19] is one of these interesting TMD parton distributions. It represents a distribution of unpolarized quarks in a transversely polarized nucleon, through a correlation between the quark's transverse momentum $\vec{k}_\perp$ and the nucleon polarization vector $\vec{S}_\perp$. The existence of the Sivers function requires final/initial-state interactions, and an interference between different helicity Fock states of the nucleon. In the absence of interactions, the Sivers function would vanish by time-reversal invariance of QCD, hence it is often referred to as a "naively time-reversal-odd" distribution. As was shown in [24, 25, 26], the interactions are represented in a natural way by the gauge link that is required for a gauge-invariant definition of a TMD parton distribution. Interference between different helicity Fock states implies

nonzero orbital angular momentum [24, 28]. Both these properties motivate the study of this function. The Sivers function will contribute to the target SSA in semi-inclusive DIS, but also to SSAs in polarized pp scattering processes such as the Drell-Yan process and di-jet or jet-photon correlations. We will discuss all these asymmetries in this paper.

The Collins function is another "naively time-reversal-odd" function. It is a transverse-momentum dependent fragmentation function and was introduced in [20]. It represents a correlation between the transverse spin of a fragmenting quark and the transverse momentum of the hadron relative to the "jet axis" in the fragmentation process. Like the Sivers function, it vanishes when integrated over all transverse momentum. Indications of a non-vanishing Collins effect have been found in semi-inclusive DIS [9]. Very recently results for measurements in e^+e^- annihilation to two hadrons have been reported, which give direct evidence for the Collins effect [30].

The formulation and study of TMD functions is really useful only when they appear in processes for which QCD factorization at small transverse momentum can be established. The processes, therefore, also need to be characterized by a large momentum scale, and there has to be additionally a small measured transverse momentum. Rigorous theoretical analyses of such reactions started from Collins and Soper's seminal paper [16], in which they proved factorization for di-hadron semi-inclusive processes in e^+e^- annihilation. Non-perturbative TMD fragmentation functions were defined and then further studied along with TMD parton distributions in [17]. The approach was extended to Drell-Yan dimuon production at hadron colliders [18].

More recently, these factorization theorems and the one for semi-inclusive DIS have been carefully (re-)examined in the context of the gauge-invariant definitions of the TMD parton distributions and fragmentation functions [31, 32], paying in particular attention to the "naively time-reversal-odd" functions. In summary, QCD factorization has been established for three classes of semi-inclusive processes: di-hadron production in e^+e^- annihilation, semi-inclusive DIS, and the Drell-Yan process. It still remains to be seen whether factorization holds for more complicated processes in hadronic scattering, such as for di-jet (di-hadron) azimuthal angular correlations [33, 34]. These processes, too, are characterized by a large scale (the individual jet or hadron transverse momenta), and by an additional small transverse momentum related, for example, to the *pair* transverse momentum, or to the deviation of the two jets from being "back-to-back". Note that this is in contrast to *single-inclusive*

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processes at hadron colliders like $p_\uparrow p \to \pi X$. The spin asymmetries for such reactions are power-suppressed ("higher twist"), and the theoretical description should be based on the methods developed in [14], where factorization in terms of higher-twist correlation functions was established.

In this paper, we will use the factorization approaches at small transverse momentum discussed above to study the single spin asymmetries in semi-inclusive DIS. We will focus on the Sivers and Collins asymmetries which presently are the most interesting ones. We realize that at the current stage it is difficult to apply the full factorization formulas developed in the literature in fitting the data and making predictions. Instead, we will make some approximations, neglecting higher order terms in the hard and soft factors. In this way we of course introduce some theoretical shortcoming, which we hope can be overcome in future studies. Our purpose is to look for the "main effects", that is, to provide a quantitative description of the spin effects now studied experimentally, and to draw our conclusions on the Collins and Sivers functions from these. Another goal of this paper is to use the information gathered from semi-inclusive DIS to make predictions for processes at RHIC, which is now taking data in transversely polarized pp scattering. With the fitted parameterizations for the Sivers functions that describe the HERMES data very well, we will calculate the Sivers asymmetries for processes at RHIC, including Drell-Yan dimuon production and di-jet and jet-plus-photon correlations. We will demonstrate that these asymmetries are expected to be large at RHIC and should therefore be closely investigated in the future polarized pp runs. This would then provide further tests of the physical picture behind the SSA, and of our theoretical understanding.

To predict the SSAs at RHIC from the distributions fitted in DIS relies on the factorization for the pp processes which, as discussed above, is so far only established for the Drell-Yan reaction. It also relies on the universality of the TMD distributions. This issue has been addressed in detail in [25, 26, 27, 32, 34]. It was found, for example, that the Sivers functions for the Drell-Yan process will have an opposite sign compared to those for SIDIS, as a result of the behavior of the gauge-links in the functions under the time-reversal operation. We will use this additional sign in our prediction for the Drell-Yan SSA at RHIC based on our Sivers function fitted to the DIS data. However, recent work has shown [34] that the issue of universality appears to be much more complicated for the case of di-jet correlations, where the more involved color structure has profound consequences on the gauge links. As a result,

the Sivers functions for this reaction will differ from those in DIS by more than just a sign. This reservation notwithstanding, in order to obtain an order of magnitude estimate we will assume in this paper that the Sivers functions to be used for di-jet correlations have the same sizes as those for the Drell-Yan processes, and opposite signs with respect to the DIS Sivers functions.

In all calculations of cross sections and asymmetries below, we will use the GRV LO parameterizations for the unpolarized quark distributions [35], and the Kretzer set of unpolarized quark fragmentation functions [36]. These will also serve as starting points for our parameterizations of the Sivers and Collins functions.

The rest of the paper is organized as follows. In Sec. II, we will review the basic formulas for the SSAs in SIDIS, and make model parameterizations for the Sivers and Collins functions. We then fit our parameterizations to the HERMES data. We will also compare our fit with the recent COMPASS data on the Sivers and Collins asymmetries. In Sec. III, we will calculate the Sivers asymmetries for the Drell-Yan process and for di-jet and jet-photon correlations at RHIC, using the fitted parameterizations from Sec. II. We summarize in Sec. IV.

II.   SSA IN SEMI-INCLUSIVE DEEP INELASTIC SCATTERING

In this section, we will study the SSA in the SIDIS processes $ep \rightarrow ehX$ and $\mu d \rightarrow \mu hX$, where h represents a hadron observed in the final state. We will compare the theoretical calculations of the asymmetries with the HERMES measurements. We will use some simple parameterizations for the Sivers functions and the Collins fragmentation functions, and fit these to the experimental data. A comparison of our fit with the COMPASS measurements will also be presented. Similar phenomenological studies of these asymmetries have also been performed in [37, 38] for the Sivers case and in [39] for the Collins asymmetry, using the earlier HERMES data.

We will start by briefly recalling the factorization formulas for the SIDIS process. For details, we refer the reader to Ref. [31]. As discussed in the introduction, we will make some simplifying approximations, in order to sharpen the constraints on the Sivers and Collins functions.

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A. Theoretical Formalism and Approximations

The differential cross section for SIDIS, including the unpolarized part and the Sivers and Collins asymmetry contributions, may be written in the following form:

$$\frac{d\sigma}{dx_B dy dz_h d^2\vec{P}_{h\perp}} = \frac{4\pi\alpha_{\rm em}^2 s}{Q^4}\left[(1-y+y^2/2)x_B\left(F_{UU}-\sin(\phi_h-\phi_S)|\vec{S}_\perp|F_{UT}^{sivers}\right)\right.$$
$$\left.-(1-y)x_B|\vec{S}_\perp|\sin(\phi_h+\phi_S)F_{UT}^{collins}\right]\ , \tag{1}$$

where ϕ_h (ϕ_S) is the angle between the lepton plane and the γ^*-hadron-plane (and the transverse target spin), y is the fraction of the incident lepton energy carried by the photon, and $\vec{P}_{h\perp}$ is the (measured) transverse momentum of the hadron. In order to compare with the experimental data, in the above formula and the following calculations, the azimuthal angles (ϕ_S and ϕ_h) are defined in the so-call virtual photon frame where the virtual photon is moving in the z direction. These definitions are different from those in [31] where a hadron frame has been chosen to define these angles. This difference has led to the different signs in the above formula, compared to that in [31]. The structure functions F_{UU} and F_{UT} will depend in general on $\vec{P}_{h\perp}$, and on the invariant mass Q^2 of the virtual photon, the Bjorken variable x_B, and on the fraction z_h of the photon longitudinal momentum carried by the hadron observed in the final state. According to the factorization formula of [31], the structure functions can be factorized into the TMD parton distributions and fragmentation functions, and soft and hard factors. For example, for the unpolarized structure function, we will have [31]

$$F_{UU}(x_B, z_h, Q^2, P_{h\perp}) = \sum_{q=u,d,s,\ldots} e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\lambda}_\perp$$
$$\times q\left(x_B, k_\perp, \mu^2, x_B\zeta, \rho\right)\hat{q}\left(z_h, p_\perp, \mu^2, \hat{\zeta}/z_h, \rho\right)S(\vec{\lambda}_\perp, \mu^2, \rho)$$
$$\times H\left(Q^2/\mu^2, \rho\right)\delta^{(2)}(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\lambda}_\perp - \vec{P}_{h\perp})\ . \tag{2}$$

This form is valid at low transverse momentum $P_{h\perp} \ll Q$ and is accurate at the leading power of $P_{h\perp}^2/Q^2$. As seen from the δ-function expressing transverse-momentum conservation, the observed hadron's transverse momentum is generated by three contributions: the transverse momentum $\vec{k}_\perp$ of partons in the nucleon, (described by the TMD distribution q), the transverse momentum $\vec{p}_\perp$ acquired in the fragmentation process (as expressed by the TMD fragmentation function $\hat{q}$), and the combined transverse momenta $\vec{\lambda}_\perp$ of (large-angle)

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soft-gluon radiation, embodied in the soft factor S. Each of these transverse momenta is integrated in Eq. (2), but leaves its imprint in the distribution in $P_{h\perp}$ of the observed hadron. In contrast to the TMD functions in (2), H is a hard factor that depends solely on the large scale Q. Furthermore, $\mu \sim Q$ is a renormalization scale, ρ a gluon rapidity cut-off parameter, and ζ is defined as $\zeta = (2P \cdot v)^2/v^2$, with P the target hadron momentum, taken in the "plus"-light-cone direction, $P = (P^+, P^- = 0, \vec{P}_\perp = \vec{0}_\perp)$, and v a time-like vector conjugate to P, i.e., with only a "minus"-light-cone component. For details, see [31], where also the related definition of $\hat{\zeta}$ is given. Similar factorization formulas as (2) can be written down for the single-transversely polarized structure functions F_{UT}^{sivers} and $F_{UT}^{collins}$.

For simplicity, in the following numerical calculations, we will use the leading order expressions for the hard scattering and the soft factors, for which we have $S = H = 1$. At this order, we may also neglect the $\zeta, \hat{\zeta}$ dependences in the parton distributions and fragmentation functions. All this brings us to the parton model picture for semi-inclusive DIS [23]. However, we stress that higher order effects can be systematically and consistently studied only within the complete factorization framework. With the above approximations, the structure functions can be simplified to [31]

$$
\begin{aligned}
F_{UU} &= \int d^2\vec{k}_\perp d^2\vec{p}_\perp \, q\,(x_B, k_\perp) \, \hat{q}\,(z_h, p_\perp) \, \delta^{(2)}(z_h\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp}) \,, \\
F_{UT}^{sivers} &= \int d^2\vec{k}_\perp d^2\vec{p}_\perp \frac{\vec{k}_\perp \cdot \hat{\vec{P}}_{h\perp}}{M} q_T\,(x_B, k_\perp) \, \hat{q}\,(z_h, p_\perp) \, \delta^{(2)}(z_h\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp}) \,, \\
F_{UT}^{collins} &= \int d^2\vec{k}_\perp d^2\vec{p}_\perp \frac{\vec{p}_\perp \cdot \hat{\vec{P}}_{h\perp}}{M_h} \delta q_T\,(x_B, k_\perp) \, \delta\hat{q}\,(z_h, p_\perp) \, \delta^{(2)}(z_h\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp}) \,, \quad (3)
\end{aligned}
$$

where a sum over all quark and antiquark flavors, weighted with the squared quark electric charge, is implicitly understood from now on. $\hat{\vec{P}}_{h\perp}$ denotes a unit vector in direction of $\vec{P}_{h\perp}$. In the above equations, q and $\hat{q}$ represent the unpolarized quark distribution and fragmentation functions, respectively, q_T the Sivers functions, $\delta\hat{q}$ the Collins functions, and δq_T the transversity distribution functions. The definitions of the above distributions and fragmentation functions are consistent with the so-called "Trento conventions" [40], while opposite in sign with respect to that used in [38] for the Sivers function. To optimize statistics, the experimental measurements of the asymmetries are normally presented after integrating over the modulus of the hadron's transverse momentum $P_{h\perp}$. After integration,

the cross section can be written as

$$\frac{d\sigma}{dx_B dy dz_h d\phi_h} = \frac{d\sigma_{UU}}{dx_B dy dz_h} - \sin(\phi_h - \phi_S)\frac{d\sigma_{UT}^{sivers}}{dx_B dy dz_h} - \sin(\phi_h + \phi_S)\frac{d\sigma_{UT}^{collins}}{dx_B dy dz_h} \ , \quad (4)$$

where the various differential cross sections will depend on x_B, z_h and y. The dependence of the cross sections on the azimuthal angles results in the azimuthal asymmetries measured in experiment. The unpolarized cross section is given by

$$\frac{d\sigma_{UU}}{dx_B dy dz_h} = \frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4}\left(1 - y + \frac{y^2}{2}\right)x_B q(x_B)\hat{q}(z_h) \ , \quad (5)$$

where $q(x_B)$ and $\hat{q}(z_h)$ are the integrated parton distribution and fragmentation functions. Here we assume that we can obtain the integrated parton distribution by integrating over the transverse momentum in the corresponding TMD parton distribution. This assumption will of course need to be modified if higher-order corrections are considered [31]. Similarly, we can calculate the polarized cross sections. In these calculations, we further assume that the final hadron's transverse momentum is entirely related to the transverse-momentum dependence in the Sivers and Collins functions. The transverse momentum contributed by the other factors in the factorized formula (2) will give some smearing effects which may be viewed as "sub-dominant". After this approximation, we can write down the polarized cross sections as

$$\frac{d\sigma_{UT}^{sivers}}{dx_B dy dz_h} = |S_\perp|\frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4}(1 - y + \frac{y^2}{2})x_B q_T^{(1/2)}(x_B)\hat{q}(z_h) \ , \quad (6)$$

$$\frac{d\sigma_{UT}^{collins}}{dx_B dy dz_h} = |S_\perp|\frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4}(1 - y)x_B \delta q_T(x_B)\delta\hat{q}^{(1/2)}(z_h) \ , \quad (7)$$

where δq_T is the integrated transversity distribution function. $q_T^{(1/2)}(x_B)$ and $\delta\hat{q}^{(1/2)}(z_h)$ are defined as

$$q_T^{(1/2)}(x_B) = \int d^2 k_\perp \frac{|\vec{k}_\perp|}{M} q_T(x_B, k_\perp) \ ,$$

$$\delta\hat{q}^{(1/2)}(z_h) = \int d^2 p_\perp \frac{|\vec{p}_\perp|}{M_h} q_T(z_h, p_\perp) \ . \quad (8)$$

The above formulas (4)-(8) will be used in the following calculations to study the experimental data for the asymmetries as functions of x_B and z_h. Before doing so, we need to set up models for the Collins and Sivers functions.

We would like to add one more comment before we proceed. In the derivation of Eq. (6), we have omitted the transverse momentum dependence of the fragmentation function, which

we referred to as "sub-dominant". This "sub-dominant" contribution could become important at small z_h where we cannot neglect the influence of the transverse momentum in the fragmentation process. For example, for a typical transverse momentum of the final state hadron of $P_{h\perp} \approx 200 \sim 300$ MeV, the quark transverse momentum could be as large as $2 \sim 3$ GeV at $z_h \approx 0.1$, much bigger than the typical value of the intrinsic quark transverse momentum for the Sivers function, which is of order of a few hundred MeV. This means that at small z_h our approximation will break down, and the transverse momentum in the fragmentation function will be important. This effect will smear out the polarized cross section and suppress the asymmetry. Indeed, when assuming a Gaussian transverse momentum dependence for both the distribution and the fragmentation functions, an additional factor of z_h appears, suppressing the polarized cross section at small z_h [38]. As a consequence, we should be cautious to apply Eq. (6) at small z_h. On the other hand, the above drawback does not apply to the case of the polarized cross section for the Collins contribution, Eq. (7), where we omit the transverse momentum dependence in the distribution. This is because there is no kinematic enhancement associated with the intrinsic transverse momentum in the parton distribution, compared to the fragmentation case.

B. Model for the Sivers functions

There exist by now quite a few model calculations for the quark Sivers functions in the nucleon [41]. The results of these vary rather widely. Here, we will instead adopt simple parameterizations for the Sivers functions and fit these to the HERMES data. We choose a form that has only a single free parameter for each flavor; the present data probably do not yet warrant a more complex form. Our parameterization is as follows:

$$
\begin{aligned}
\frac{u_T^{(1/2)}(x)}{u(x)} &= S_u x(1-x) \; , \\
\frac{d_T^{(1/2)}(x)}{u(x)} &= S_d x(1-x) \; ,
\end{aligned}
\tag{9}
$$

where in both equations $u(x)$ is the unpolarized u-quark distribution. We assume that only the quark Sivers functions are non-zero, and that the antiquark ones vanish. This assumption will of course likely need to be modified at small x. In the above parameterization, the factor x on the right-hand-side represents the valence nature of the Sivers function, whereas the

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factor $(1 - x)$ denotes an expected suppressed behavior of the function at large x [1]

C.   Model for the Collins functions

For the Collins asymmetry, we have two sets of unknown functions: the transversity distributions and the Collins fragmentation functions. From the present experimental data on the Collins asymmetries we cannot obtain constraints on both of them simultaneously [2]. Here we adopt a parameterization for the transversity function [42] that is based on saturation of the Soffer inequality [43]. We note that this parameterization represents an upper bound for the transversity functions for the quarks.

As for the Sivers functions, there have also been several model calculations for the Collins functions [44], showing rather wide variations. Again, we will just use a simple parameterization for the z_h dependence of $\delta\hat{q}^{(1/2)}$. The flavor dependence of the Collins functions is important since one would like to describe the asymmetries for different hadron species. From the theory side, one could get constraints for the flavor dependence based on momentum conservation in the fragmentation process: the Schäfer-Teryaev sum rules [45]. These sum rules state that the odd-moment (for the intrinsic transverse momentum) of the Collins function vanishes when the function is summed over all hadron states. Because the sum rules only involve the integrals over z_h and $p_\perp$ (with a weight $p_\perp$) of the Collins functions, one cannot obtain from them more detailed constraints, e.g., for the z_h or $p_\perp$ dependences. In the following we will motivate a simple conjecture for the Collins functions, based on quark-hadron duality in the fragmentation process. This will provide us with additional constraints. The main result is that *any quark Collins fragmentation function is very small when summed over all final hadrons.* For example, the u quark Collins functions to all hadron final states will satisfy

$$\sum_h \delta\hat{u}^h(z_h, p_\perp) = \mathcal{O}(m_u) \ . \tag{10}$$

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[1] This power suppression in $(1 - x)$ could actually be as strong as $(1 - x)^2$ [37]. In our fit, most data are in the intermediate range of x, and the $x \to 1$ limit is not really reached. In any case, the power of $(1 - x)$ will be modified by logarithms.

[2] Note, however, that independent information on the Collins functions is now coming from measurements of hadron-pair production by the BELLE collaboration [30]. It is hoped that combination of these results with those from lepton scattering would eventually give information on transversity.

The above equation is motivated as follows. The Collins function is defined as

$$\frac{\epsilon^{ij} p_\perp^j}{M_h} \delta \hat{q}^h(z_h, p_\perp) = \frac{n^-}{3z} \int \frac{d\xi^+}{2\pi} \frac{d^2\vec{b}}{(2\pi)^2} e^{-i(k^-\xi^+ - \vec{k}_\perp \cdot \vec{b}_\perp)} \sum_X \text{Tr} \left\{ \gamma_5 \gamma^- \gamma^i \right. \tag{11}$$

$$\left. \times \langle 0 | \mathcal{L}(-\infty, 0) \psi(0) | P_h X \rangle \langle P_h X | \overline{\psi}(\xi^+, \vec{b}) \mathcal{L}^\dagger(\xi^+, \vec{b}, -\infty) | 0 \rangle \right\} \,,$$

where $k^- = P_h^- / z_h$ and $\vec{k}_\perp = -\vec{p}_\perp / z_h$. Here, the final state hadron has been taken to have a large light-cone "minus" momentum component P_h^- [we remind the reader that P_h denotes the momentum of the observed hadron, while $\vec{p}_\perp$ is the (integrated) transverse momentum that the hadron acquires in the fragmentation process relative to the fragementing parton; see Eq. (2)]. Finally, $\mathcal{L}$ is the gauge link along the light-cone direction conjugate to P_h. Here, Now, if we sum over all hadrons in the above Collins fragmentation functions, the intermediate hadronic states can be replaced by a quark or a quark plus gluons (or quark-antiquark pairs) using quark-hadron duality arguments. We then get the following equation:

$$\sum_h \frac{\epsilon^{ij} p_\perp^j}{M_h} \delta \hat{q}^h(z_h, p_\perp) = \frac{n^-}{3z} \int \frac{d\xi^+}{2\pi} \frac{d^2\vec{b}}{(2\pi)^2} e^{-i(k^-\xi^+ - \vec{k}_\perp \cdot \vec{b}_\perp)} \sum_X \text{Tr} \left\{ \gamma_5 \gamma^- \gamma^i \right. \tag{12}$$

$$\left. \times \langle 0 | \mathcal{L}(-\infty, 0) \psi(0) | P_q X \rangle \langle P_q X | \overline{\psi}(\xi^+, \vec{b}) \mathcal{L}^\dagger(\xi^+, \vec{b}, -\infty) | 0 \rangle \right\} \,,$$

whose validity rests on the argument of quark-hadron duality for the fragmentation process. Duality-breaking effects will somewhat modify the above equation. The right hand side of Eq. (12) may be viewed as a quark Collins fragmentation function into a quark (or antiquark/gluon) state. The helicity-flip required for a non-vanishing Collins function is then possible because of a finite quark mass. Thus, we approximately expect

$$\sum_h \delta \hat{q}^h(z_h, p_\perp) = \mathcal{O}(m_q) \approx 0 \,. \tag{13}$$

If we further assume that the fragmentation functions for u and d quarks to strange mesons are suppressed relative to those into pions, we can have even stronger constraints for the pion Collins functions:

$$\delta \hat{q}^{\pi^+}(z_h, p_\perp) + \delta \hat{q}^{\pi^-}(z_h, p_\perp) + \delta \hat{q}^{\pi^0}(z_h, p_\perp) \approx 0 \,, \tag{14}$$

where q represents any flavor of u, d quarks and their antiquarks. Further simplification of the above equation can be derived by considering isospin and charge symmetry relations between the different fragmentation functions. For example, we will have the following

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relations,

$$\delta \hat{u}^{\pi^+} = \delta \hat{d}^{\pi^-} = \delta \hat{\bar{d}}^{\pi^+} = \delta \hat{\bar{u}}^{\pi^-} \equiv \delta \hat{q}^{\pi}_{fav.}$$

$$\delta \hat{d}^{\pi^+} = \delta \hat{u}^{\pi^-} = \delta \hat{\bar{u}}^{\pi^+} = \delta \hat{\bar{d}}^{\pi^-} \equiv \delta \hat{q}^{\pi}_{unfav.}$$

$$\delta \hat{u}^{\pi^0} = \delta \hat{d}^{\pi^0} = \delta \hat{\bar{d}}^{\pi^0} = \delta \hat{\bar{u}}^{\pi^0} = \frac{1}{2} \left[\delta \hat{q}^{\pi}_{fav.} + \delta \hat{q}^{\pi}_{unfav.} \right] \ . \tag{15}$$

Here $\delta \hat{q}^{\pi}_{fav.}$ and $\delta \hat{q}^{\pi}_{unfav.}$ represent the "favored" (in the sense that the leading Fock state of the hadron contains the parent quark flavor) and "unfavored" (where it does not contain it) fragmentation functions, respectively. Substituting the above relations into Eq. (14), we will obtain

$$\delta \hat{q}^{\pi}_{fav.} + \delta \hat{q}^{\pi}_{unfav.} \approx 0 \ , \tag{16}$$

which means that the unfavored Collins function is approximately equal to the favored one with opposite sign. This result is counter to the usual notion in the literature that the favored fragmentation functions should be much larger than the unfavored ones. It of course crucially depends on the validity of the approximations made in the above derivation and will be subject to some corrections. We note, however, that this observation has also support from a string model description for the Collins fragmentation function [46]. As a test, we will treat in the following the favored and unfavored Collins functions free from the constraint (16), and fit them to the data to see if they naturally satisfy the above relation or not. To parameterize the Collins functions, we use the following two sets of functional forms,

$$\text{Set I}: \quad \delta \hat{q}^{\pi(1/2)}_{fav.}(z) = C_f z(1-z) \hat{u}^{\pi^+}(z) \ , \quad \delta \hat{q}^{\pi(1/2)}_{unfav.}(z) = C_u z(1-z) \hat{u}^{\pi^+}(z) \ ,$$

$$\text{Set II}: \quad \delta \hat{q}^{\pi(1/2)}_{fav.}(z) = C_f z(1-z) \hat{u}^{\pi^+}(z) \ , \quad \delta \hat{q}^{\pi(1/2)}_{unfav.}(z) = C_u z(1-z) \hat{d}^{\pi^+}(z) \ . \tag{17}$$

The z factor in these parameterizations represents the vanishing of the Collins function at small z, and the $(1-z)$ factor follows arguments made in [20]. The difference between these two sets is that for Set I we parameterize both favored and unfavored Collins functions in terms of the favored unpolarized quark fragmentation function, while for Set II we parameterize the unfavored Collins function using the unfavored unpolarized quark fragmentation function. Set I is inspired by the constraint of Eq. (16); note that this ansatz is expected to violate the positivity constraints at very large z_h. On the other hand, Set II respects the positivity constraints, provided $|C_{f,u}| \leq 4$. In the following, we will fit the HERMES data with these two sets of parameterizations.

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D. Comparison with SIDIS Data

We will now calculate the Collins and Sivers asymmetries using the functions specified above, and fit the free parameters to the new HERMES data on the Collins and Sivers asymmetries [10]. Here we will use $Q^2 = 2.41 \text{ GeV}^2$, which is the average for the HERMES kinematics. We choose $\mu = Q$ in the unpolarized parton distribution and fragmentation functions. We will then compare our fit results to the recent measurements by COMPASS [12]. For the fitting, we use the CERNLIB MINUIT routine [47].

As a function of x_B, the Sivers asymmetry can be calculated from the following formula:

$$A_N(x_B) = -\frac{\int dz_h dy \frac{d\sigma_{UT}^{sivers}}{dx_B dy dz_h}}{\int dz_h dy \frac{d\sigma_{UU}}{dx_B dy dz_h}} \ , \tag{18}$$

where the minus sign results from the minus sign in the polarized differential cross section in Eq. (4). Since the y integral is the same for the numerator and denominator, it cancels out. Moreover, the integral over z_h can be factored out as a consequence of the approximations that led to Eqs. (5) and (6), and the Sivers asymmetry will be proportional just to the ratio of the Sivers functions over the unpolarized quark distributions, summed appropriately over flavors.

The Sivers asymmetry as a function of z_h can be calculated similarly. In Fig. 1, we show the results of our fit of the Sivers asymmetries for π^+ and π^- to the HERMES data [10]. For the two free parameters the fit gives

$$S_u = -0.81 \pm 0.07, \quad S_d = 1.86 \pm 0.28 \ , \tag{19}$$

with $\chi^2/\text{d.o.f} \approx 1.2$. The band in each plot of Fig. 1 corresponds to a 1-σ error in the determined parameters. One can see that the u quark Sivers function appears somewhat better constrained by the data than the d quark one. This is readily understood from the fact that u quarks in DIS enter with the charge factor 4/9, so that in scattering off a proton target the u quark distribution is particularly selected. Another feature is that the d Sivers function comes out larger (by about a factor of two) than the u quark one, and with opposite sign. This behavior is quite different from model calculations [41]. The result is due to the fact that the HERMES Sivers asymmetry for π^- is much smaller than that for π^+. Theoretically, however, π^- production should also have a significant contribution from u quarks, because one finds $\hat{u}^{\pi^-} \approx 0.6\hat{u}^{\pi^+}$ for the fragmentation functions when integrated
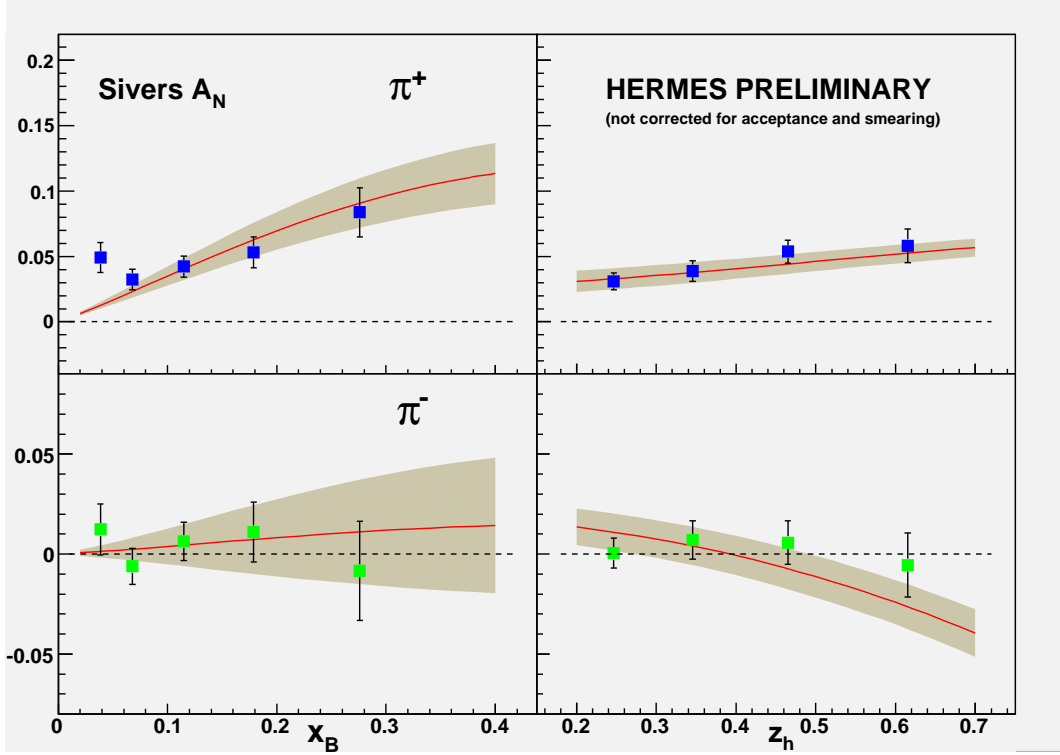
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FIG. 1: *Sivers SSA fit to the HERMES data [10]; see text. The bands correspond to the 1-σ error of the fitted parameters. Note that the data have not yet been corrected for acceptance and smearing.*

over the experimentally relevant region $0.2 < z < 0.7$. To obtain a much smaller asymmetry for π^- than for π^+, there then have to be fairly strong cancellations between the u and d quark Sivers functions. We note that the signs we find for our Sivers functions are consistent with expectations in [29], where they were qualitatively related to the opposites of the quark contributions to the proton anomalous magnetic moment.

Figure 2 shows predictions for the π^0 Sivers asymmetries as functions of x_B and z_h, based on our fits for the Sivers functions. We note that our prediction for the π^0 asymmetry is nearly independent of z_h. This is because the u and d quark fragmentation functions for π^0 are the same, and because in our approximation the distribution and fragmentation functions are decoupled. However, we have to keep in mind that this decoupling might break down at small z_h, as we discussed before. This could be tested by future HERMES data.

In [37, 38], earlier HERMES results [9] for the Sivers asymmetries were fitted. The methods somewhat differed from ours. In [38], a particular transverse momentum dependence is assumed for the Sivers functions and the unpolarized quark distribution and fragmentation
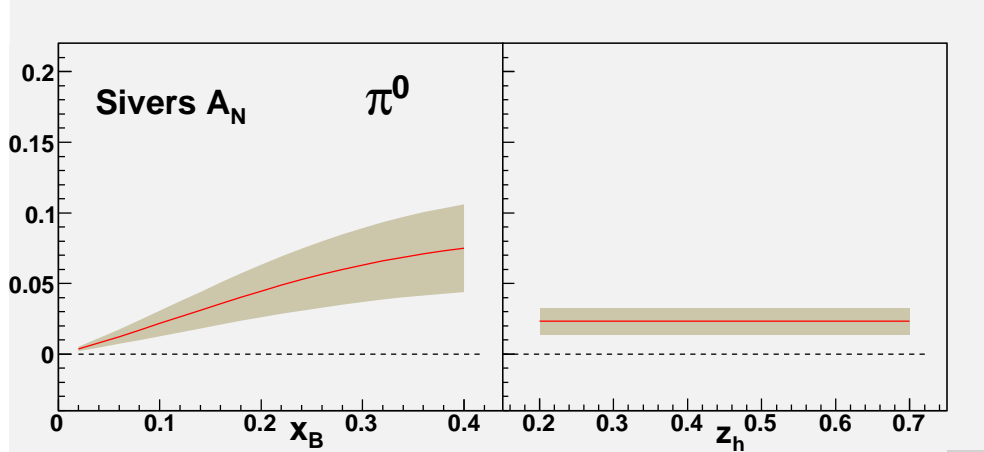
FIG. 2: *Predicted Sivers SSA asymmetries for π^0 production at HERMES.*

functions. With more free parameters the experimental data are fitted equally well, and the u and d valence Sivers functions are obtained from the fit. In [37], the asymmetries weighted with the transverse momentum of the hadron were used for the fit. Both fits find a large d quark Sivers function with opposite sign relative to the u-quark one. We have also checked that our fit results for the Sivers functions are consistent with these fits within the current large uncertainties, where we notice that the Sivers function in [38] has an opposite sign compared to ours and to the "Trento conventions" [40].

The COMPASS collaboration also has measured the Sivers asymmetry [12], separately for positively and negatively charged hadrons, produced off a deuteron target. To simplify the comparison with their data, we assume that the leading hadrons are mostly pions. We calculate the Sivers asymmetries for π^+ and π^- in the kinematic region of the COMPASS experiment, using the above fitted Sivers functions for u and d quarks, and compare to their data for leading positive and negative hadrons, respectively. We show this comparison in Fig. 3. One can see that our calculations based on fits to the HERMES data are also consistent with the COMPASS data, within error bars. We note that for the kinematical region of the COMPASS experiment, our predicted Sivers asymmetries for a deuteron target are very small, except in the large-x valence region. The smallness of the Sivers asymmetry is again related to cancellations between u and d contributions, which for deuterons enter in a different combination than for a proton target. It will be very interesting to check these predictions with future COMPASS data for a proton target. Thanks to the higher Q^2, such data would also help in confirming the leading-twist nature of the Sivers and Collins
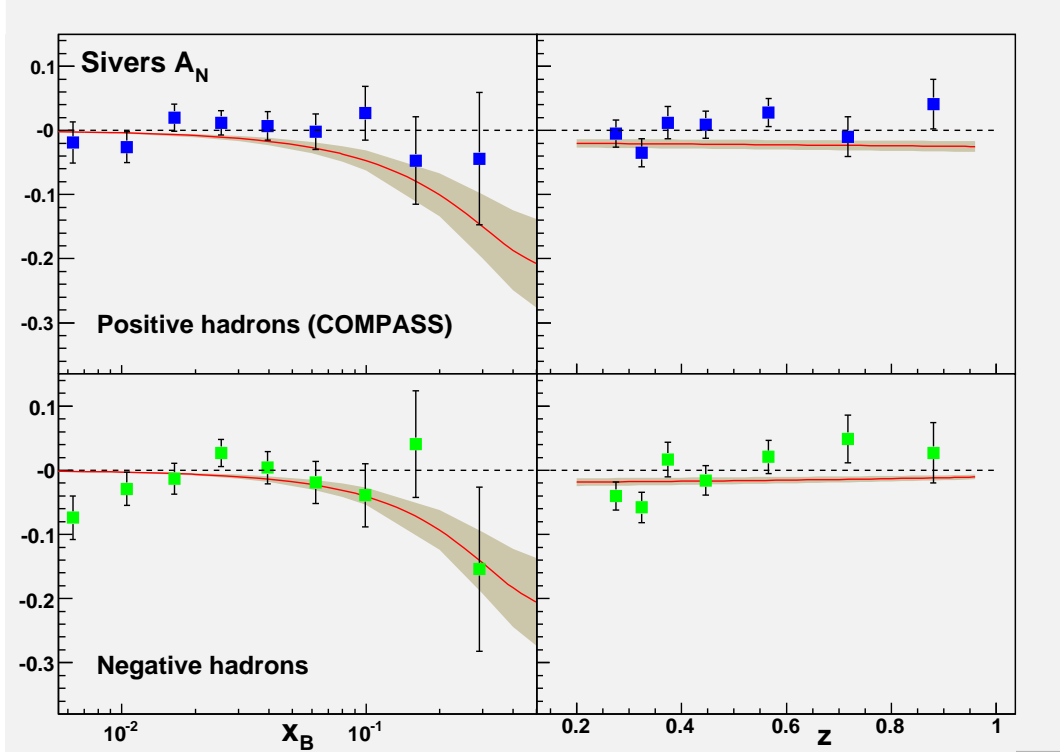
FIG. 3: *Sivers asymmetries compared to the COMPASS data [12].*

asymmetries.

We next turn to the Collins asymmetry. Here we follow a similar procedure as we did for the Sivers case above. As we mentioned earlier, the situation is more complicated because of the fact the nucleon transversity densities are currently not known, and we need to resort to a model or ansatz for the latter. As described above, we will use the parameterizations for the quark transversity distributions of [42], which represent upper bounds for the densities. We will fit to the HERMES data using the two sets of simple parameterizations for favored and unfavored Collins functions given in Eq.(17).

The asymmetry as a function of x_B is calculated from the formula

$$A_N^h(x_B) = -\frac{\sum\limits_{q=u,d} e_q^2 \delta\hat{q}^{(1/2)h} \int dy \frac{1-y}{x_B^2 y^2} x_B \delta q_T(x_B)}{\sum\limits_{q=u,d,\bar{u},\bar{d}} e_q^2 \hat{q}^h \int dy \frac{1-y+y^2/2}{x_B^2 y^2} x_B q(x_B)} \ , \qquad (20)$$

where again the minus sign comes from the sign in the polarized differential cross section, Eq. (4). $\delta\hat{q}^{(1/2)h}$ and $\hat{q}^h$ represent the fragmentation functions integrated over the accessed region in z_h. Kinematic cuts impose a correlation between x_B and y, and the integral over y will depend on x_B. In the experimental analysis, the data for the Collins asymmetries
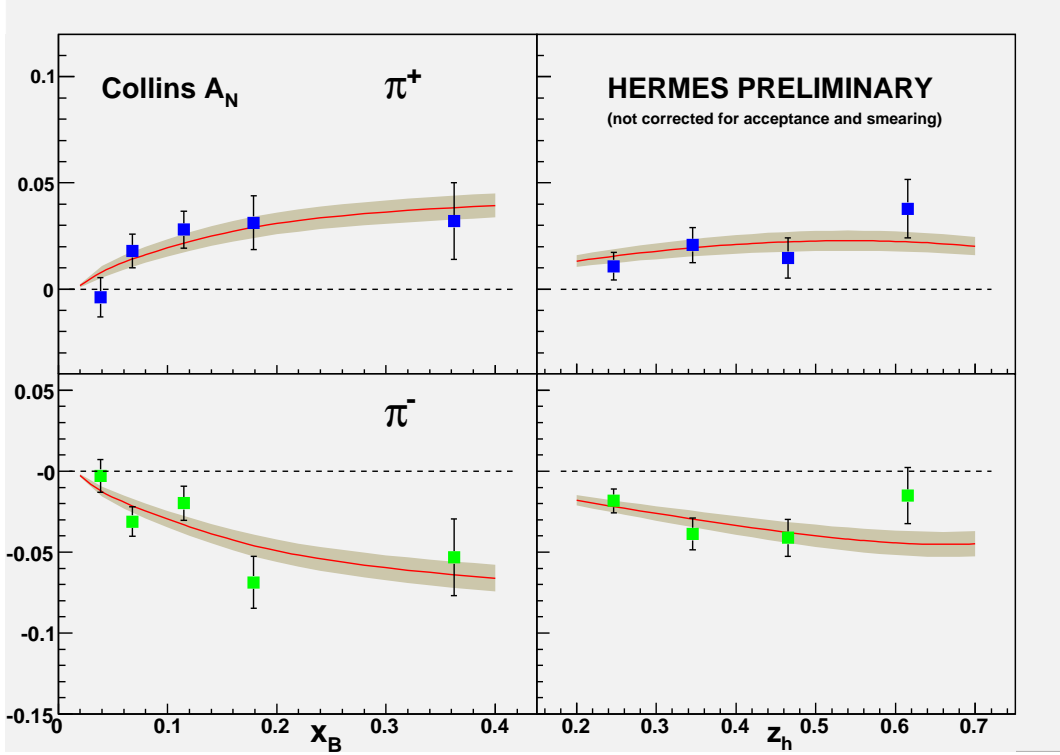
FIG. 4: *Same as Fig. 1 for the Collins asymmetries, using the Set I parameterization of the Collins functions. The data [10] and the theory curves are for the so-called lepton beam asymmetries. Note that the data have not yet been corrected for acceptance and smearing.*

are presented in two different ways. One is to give results in terms of the virtual-photon asymmetry, factoring out the term $(1 - y)/(1 - y + y^2/2)$. The other way is to give the directly measured lepton-beam asymmetry. In our calculations, we follow the latter way. We neglect the contribution of longitudinal photons to the unpolarized cross section, which HERMES has considered in the analysis of the virtual-photon asymmetries [10]. In view of the overall uncertainties, this is a minor effect, as we have checked by comparing also to the virtual-photon asymmetries. From the fit to the lepton-beam asymmetry data, we get the two fit parameters as follows:

$$\text{Set I}: \qquad C_f = -0.29 \pm 0.04, \quad C_u = 0.33 \pm 0.04 , \qquad (21)$$

$$\text{Set II}: \qquad C_f = -0.29 \pm 0.02, \quad C_u = 0.56 \pm 0.07 , \qquad (22)$$

with $\chi^2/\text{d.o.f.} \approx 0.8(0.7)$ for the Set I and Set II parameterizations, respectively. The fit results are shown in Figs. 4 and 5, compared to the HERMES data. Both fits are of the same quality.
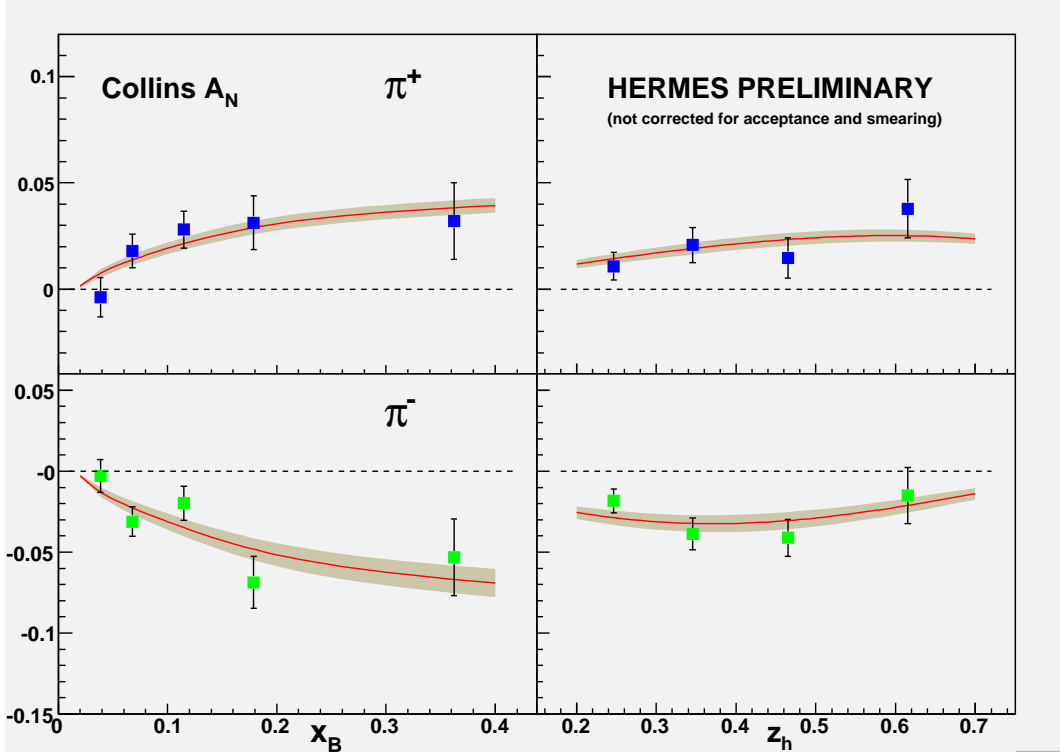
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FIG. 5: *Same as Fig. 4, for the Set II parameterizations for the Collins functions.*

In Figs. 6 and 7, we plot the fitted favored and unfavored Collins functions (times z) for Sets I and II respectively. Note that we multiply the favored ones by (-1) to compare their magnitudes. For comparison, we also show the corresponding unpolarized quark fragmentation functions [36]. It is evident that the two sets of Collins functions indeed both satisfy the positivity constraints. The equal quality of the fits obtained for sets I and II implies that the current experimental data neither necessarily support the constraints we derived in Eq. (16), nor do they rule them out. However, from both fits we indeed find that in a quite large range of z_h the unfavored Collins function has the same size as that of the favored one with opposite sign. A similar conclusion was obtained from a fit to this asymmetry using the transversity functions calculated in the Chiral Quark Model [39]. We hope that higher-statistics data will become available in the near future that will test the relations.

Figures 8 and 9 show predictions for the π^0 Collins asymmetries as functions of x_B and z_h, based on our fits for the Collins functions. From these plots, we find that the asymmetries are very small for both sets of the Collins functions, because of strong cancellations between the contributions from favored and unfavored Collins functions. We note that preliminary
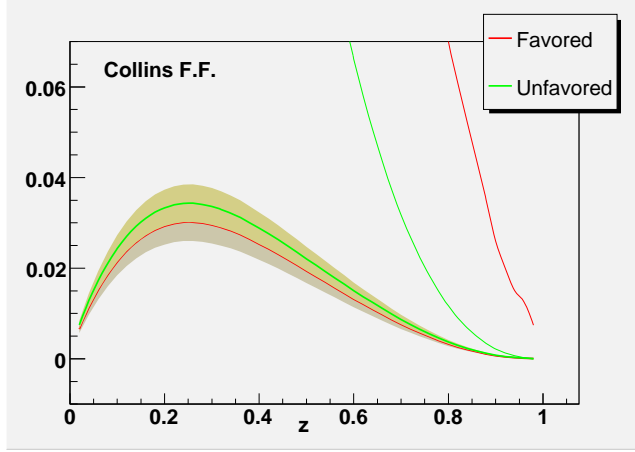
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FIG. 6: *Set I* $(-1)\times$*favored and unfavored Collins fragmentation functions. Also shown are the unpolarized quark fragmentation functions from Kretzer's parameterizations.*
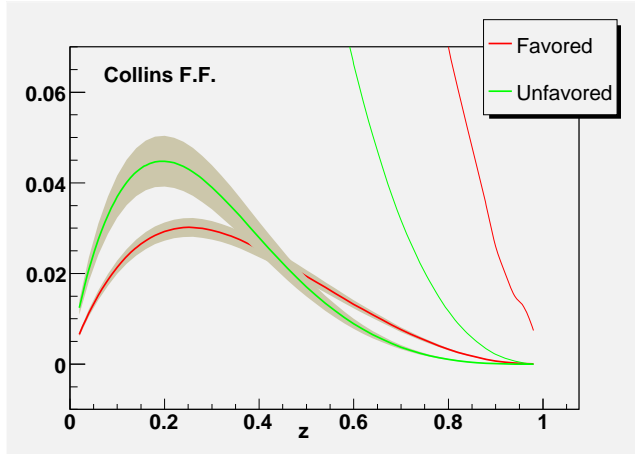

FIG. 7: *Same as Fig. 6 for the Set II Collins fragmentation functions.*

data from HERMES indeed indicate that the π^0 asymmetry is consistent with zero [48]. Higher-statistics data on this will be highly interesting for testing the above conclusions.

As before, we also compare our fit to the COMPASS measurements [12] for the Collins asymmetries, where only the virtual-photon asymmetries are presented. We also note that the convention for the Collins asymmetry used by the COMPASS Collaboration is different from that used by HERMES. In our calculations, we have made the relevant modifications in order to compare with the COMPASS data. We show these comparisons in Figs. 10 and 11 for our Set I and II Collins functions, respectively. As for the Sivers case above, there is good consistency. The overall asymmetries are again small because of the deuteron
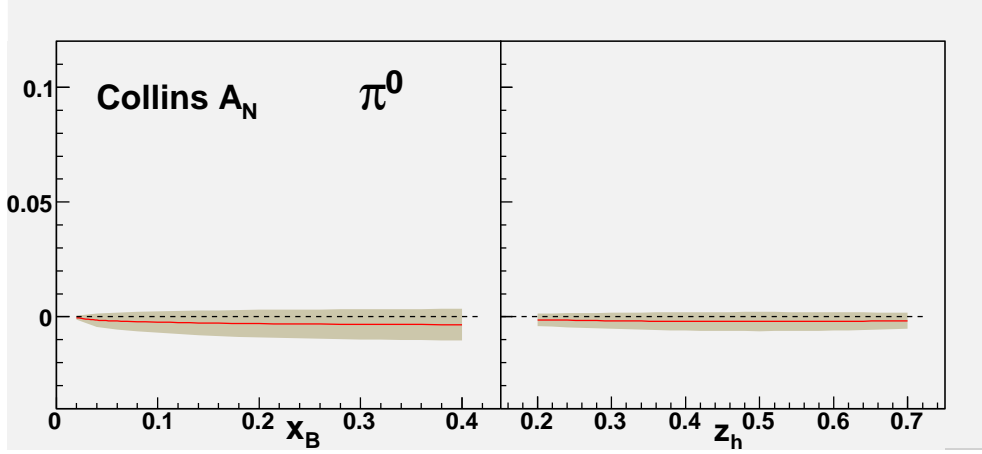
FIG. 8: *Predicted Collins SSA asymmetries for π^0 production at HERMES with Set I.*
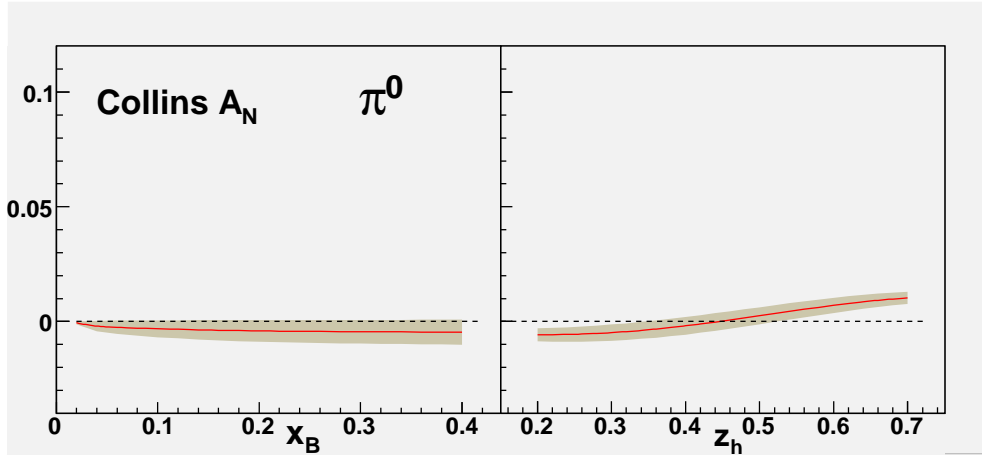

FIG. 9: *Predicted Collins SSA asymmetries for π^0 production at HERMES with Set II.*

target used, and because the assumed transversity sea quark distributions are small. Future high-statistics COMPASS data for a proton target would be highly interesting.

III.   SINGLE-TRANSVERSE SPIN ASYMMETRIES AT HADRON COLLIDERS

An important issue in the study of hard-scattering processes is the universality of the nonperturbative objects: the parton distributions and fragmentation functions. In the case of single-spin asymmetries, if universality holds, the Sivers functions obtained from, for example, SIDIS can be used to predict single-spin asymmetries in pp or $\bar{p}p$ scattering. The universality between different classes of processes is a complicated and interesting issue that has attracted much interest recently [32, 34, 49, 50]. As we described earlier, it was found
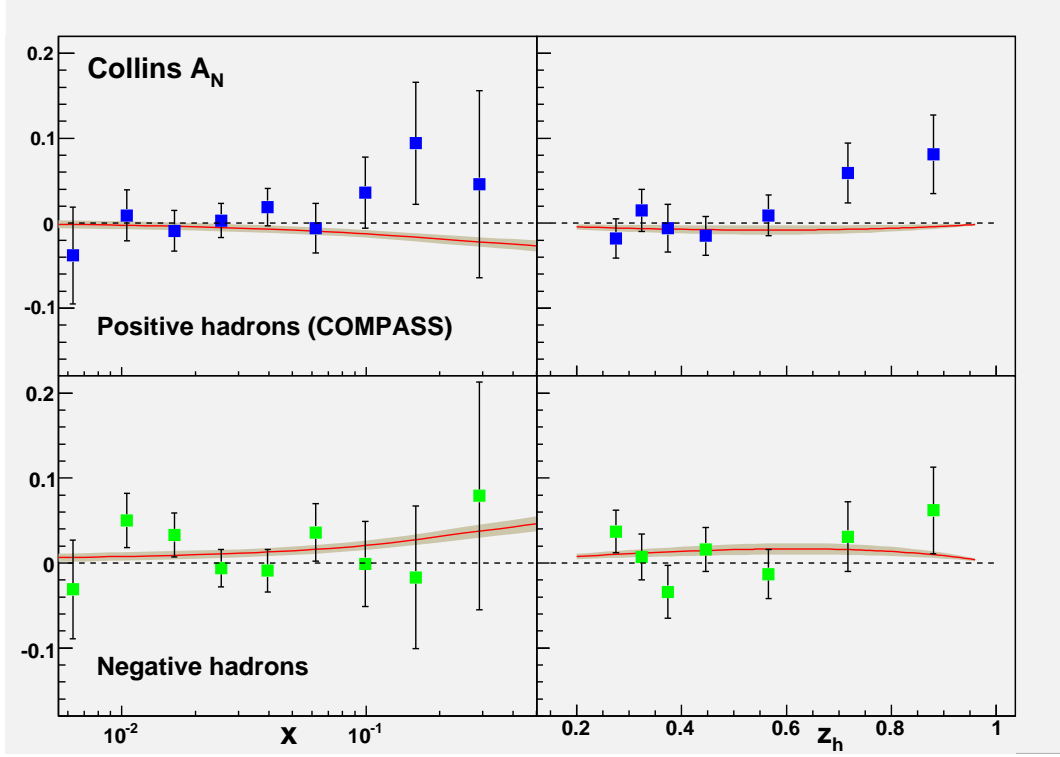
FIG. 10: *Collins asymmetry compared with COMPASS data [12] for Set I of our fitted Collins functions.*

that, while strict universality is violated already when going from SIDIS to the Drell-Yan process, the Sivers functions for the two processes differ only by a sign. It is therefore possible to use the fitted Sivers function from the last section and to predict the Sivers single-spin asymmetry for the Drell-Yan process at hadron colliders. More complicated processes in hadronic scattering, such as the SSA in di-jet angular correlations [33], are not yet completely understood at present, as far as factorization and universality are concerned. Progress has been made recently [34]; it appears that similarly to the Drell-Yan process universality is violated only by terms that are calculable from the color structure of the partonic scattering and hence may be taken into account in phenomenological analyses. Below, we will also give estimates for Sivers contributions to SSAs in di-jet correlations and in jet-plus-photon correlations at RHIC, using the Sivers functions of Section II, and the usual unpolarized hard-scattering functions. In the light of Ref. [34], we expect that our estimates will likely need to be revised once these reactions will be completely understood in the context of factorization and universality, to take into account the appropriate factors
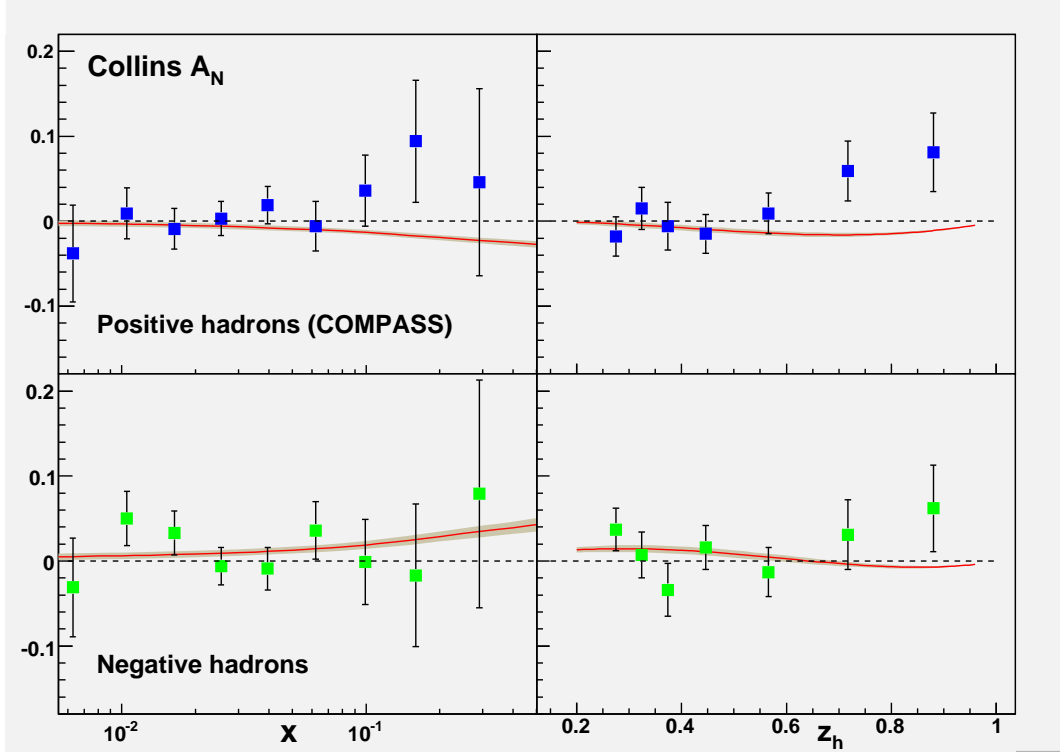
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FIG. 11: *Same as Fig. 10, but for Set II of Collins fragmentation functions.*

embodying the nonuniversality of the Sivers functions. We shall briefly return to this point below.

A.   Drell-Yan Dimuon Production $p^\uparrow p \to \mu^+ \mu^- X$

In this subsection, we will calculate the Sivers single-spin asymmetry for the Drell-Yan process at RHIC, using the fit result of the last section [see Eqs. (9) and (19)]. As just discussed, one has

$$q_T^{DY} = -q_T^{DIS} \ . \tag{23}$$

After integrating out the lepton angles in the rest frame of the virtual photon, we obtain the following differential cross section for the Drell-Yan process:

$$\frac{d\sigma}{dM^2 dy d^2 q_\perp} = \frac{4\alpha^2 \pi}{3sM^2} \left[W_0(x_1, x_2, M^2, q_\perp) + \sin\phi \, W_{TU}(x_1, x_2, M^2, q_\perp) \right] \ , \tag{24}$$

where M is the invariant mass of the lepton pair, $q_\perp$ the virtual photon's transverse momentum, and y its rapidity. $\phi = \phi_{\gamma^*} - \phi_S$ is the difference between the azimuthal angles of the virtual photon and the transverse polarization vector in a frame where the polarized

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hadron is moving in the z direction. At low transverse momentum, x_1 and x_2 are related to the mass and rapidity through $x_1 = M/\sqrt{s}\,e^y$ and $x_2 = M/\sqrt{s}\,e^{-y}$ where s is the hadronic center-of-mass energy squared. According to the factorization theorem [31], the hadronic tensors W_0 and W_{UT} can be factorized into the TMD parton distributions, and soft and hard-scattering factors. Again, neglecting the soft factor and using the Born expression for the hard part, we obtain simple expressions for the tensors:

$$
W_0 = \sum_q \frac{e_q^2}{3} \int d^2k_{1\perp} d^2k_{2\perp} q(x_1, k_{1\perp}) \overline{q}(x_1, k_{2\perp}) \delta^{(2)}(\vec{k}_{1\perp} + \vec{k}_{2\perp} - \vec{q}_\perp) ,
$$

$$
W_{TU} = \sum_q \frac{e_q^2}{3} \int d^2k_{1\perp} d^2k_{2\perp} \frac{\vec{k}_{1\perp} \cdot \hat{\vec{q}}_\perp}{M} q_T(x_1, k_{1\perp}) \overline{q}(x_1, k_{2\perp}) \delta^{(2)}(\vec{k}_{1\perp} + \vec{k}_{2\perp} - \vec{q}_\perp) , \quad (25)
$$

where $q_T(x_1, k_{1\perp})$ is now the Sivers function for the Drell-Yan process. A further approximation can be made by integrating out the transverse momentum $|\vec{q}_\perp|$, but keeping the dependence on azimuthal angle. The differential cross section can then be written as

$$
\frac{d\sigma}{dM^2 dy d\phi} = \frac{4\alpha^2\pi}{3sM^2} \left[\tilde{W}_0(x_1, x_2, M^2) + \sin\phi \tilde{W}_{TU}(x_1, x_2, M^2) \right] , \quad (26)
$$

where

$$
\tilde{W}_0 = \sum_q \frac{e_q^2}{3} q(x_1) \overline{q}(x_2) , \quad (27)
$$

and

$$
\tilde{W}_{UT} = \sum_q \frac{e_q^2}{3} q_T^{(1/2)}(x_1) \overline{q}(x_2) . \quad (28)
$$

In Fig. 12, we plot the $\sin\phi$ asymmetries as functions of the photon rapidity y and the invariant mass M. From this plot, we see that the Sivers SSA asymmetry for the Drell-Yan process at RHIC is expected to be sizable for large rapidity, and should be measurable at RHIC if enough statistics can be accumulated in transverse-spin running. We note that the Sivers asymmetry in the Drell-Yan process is also a particular focus for proposed measurements in polarized $\overline{p}p$ scattering at the planned GSI-FAIR facility [37, 51, 52].

B.   Correlations in $p^\uparrow p \rightarrow \text{jet}_1(\vec{P}_{1\perp}) + \text{jet}_2(\vec{P}_{2\perp}) + X$

Other interesting observables at hadron colliders from which one can access the intrinsic transverse-momentum dependence of parton distributions are "back-to-back" correlations between two jets [33, 34]. More specifically, we are interested in situations in which the
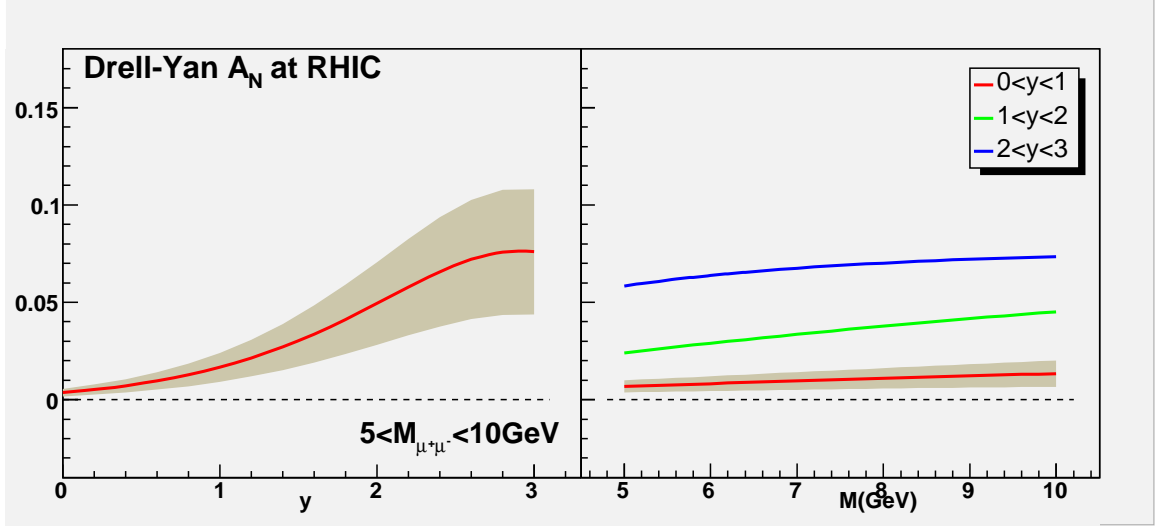
FIG. 12: *Sivers asymmetries for the Drell-Yan process at RHIC, as functions of virtual-photon rapidity y and invariant mass M.*

sum of the two jet transverse momenta, $\vec{q}_\perp \equiv \vec{P}_{1\perp} + \vec{P}_{2\perp}$ (or a component or projection thereof), is measured, while both $P_{1\perp}$ and $P_{2\perp}$ individually are large. As for the Drell-Yan process discussed above, this is usually a small transverse momentum, much smaller than the large scales $|\vec{P}_{1\perp}| \approx |\vec{P}_{2\perp}|$ characterizing the overall process, and a special factorization may apply. Let us first, however, consider the cross section integrated over $\vec{q}_\perp$. Here, collinear factorization applies, and the di-jet cross section has the parton-model expression [53],

$$\frac{d\sigma}{dy_1 dy_2 dP_\perp^2} = \sum_{ab} x_a f_a(x_a) x_b f_b(x_b) \frac{d\hat{\sigma}}{d\hat{t}}(ab \rightarrow cd) \ , \tag{29}$$

where $d\hat{\sigma}/d\hat{t}$ is the differential cross section for the partonic process $ab \rightarrow cd$, with $f_{a,b}$ the appropriate parton distribution functions. We have defined the transverse momentum $P_\perp \equiv |\vec{P}_{1\perp}| = |\vec{P}_{2\perp}|$, and y_1 and y_2 denote the rapidities of the two jets. The kinematics are as follows:

$$x_a = \frac{P_\perp}{\sqrt{s}} \left(e^{y_1} + e^{y_2} \right), \quad x_b = \frac{P_\perp}{\sqrt{s}} \left(e^{-y_1} + e^{-y_2} \right)$$

$$\hat{s} = x_a x_b s, \quad \hat{t} = -P_\perp^2 \left(e^{y_2 - y_1} + 1 \right), \quad \hat{u} = -P_\perp^2 \left(e^{y_1 - y_2} + 1 \right) \ . \tag{30}$$

Here, $\hat{s}$, $\hat{t}$, and $\hat{u}$ are the usual partonic Mandelstam variables appearing in the partonic cross sections for the reactions $ab \rightarrow cd$, namely $\hat{s} = (p_a + p_b)^2$, $\hat{t} = (p_a - p_c)^2$, $\hat{u} = (p_a - p_d)^2$, in obvious notation of the partonic momenta. The leading order contributions produce the

di-jet pair exactly balanced, that is back-to-back in the partonic center-of-mass frame. An imbalance in the transverse direction is generated by higher-order QCD corrections. At small but nonzero imbalance between the two jets, the dominant contributions will come from the intrinsic transverse momenta of the initial partons. As a model, we will generalize the above factorization formula to the case of small $\vec{q}_\perp$, in analogy with the SIDIS and Drell-Yan cases discussed earlier, taking into account the various contributions to the transverse momentum dependence coming from the parton distributions and soft factors:

$$
\frac{d\sigma}{dy_1 dy_2 dP_\perp^2 d^2\vec{q}_\perp} = \sum_{ab} \int d^2 k_{1\perp} d^2 k_{2\perp} d^2 \lambda_\perp x_a f_a(x_a, k_{1\perp}) x_b f_b(x_b, k_{2\perp})
$$
$$
\times S_{ab\to cd}(\lambda_\perp) H_{ab\to cd}(P_\perp^2) \delta^{(2)}(\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{\lambda}_\perp - \vec{q}_\perp) \ , \tag{31}
$$

where $S_{ab\to cd}$ is a soft factor for the process $ab \to cd$, while $H_{ab\to cd}$ is the hard part of the reaction, related to lowest order to $d\hat\sigma/d\hat t$. We emphasize the overly simplistic character of Eq. (31) as it stands. The detailed factorized form (if it exists) will likely be different; in particular, one expects an interplay of the color structures of the soft factors and the hard parts, as found in resummation studies for jet cross sections [54].

In a similar fashion, we write the Sivers-type contribution $d\sigma_{TU}$ to the single-polarized cross section,

$$
\frac{d\sigma}{dy_1 dy_2 dP_\perp^2 d^2\vec{q}_\perp} = d\sigma_{UU} + \vec{e}_z \cdot \left(\vec{S}_\perp \times \hat{\vec{q}}_\perp\right) d\sigma_{TU} \ , \tag{32}
$$

where $\vec{e}_z$ is the unit vector in the z-axis direction. In a factorized form, we will get:

$$
d\sigma_{TU} = \sum_{ab} \int d^2 k_{1\perp} d^2 k_{2\perp} d^2 \lambda_\perp \frac{\vec{k}_{1\perp} \cdot \hat{\vec{q}}_\perp}{M} x_a q_{Ta}(x_a, k_{1\perp}) x_b f_b(x_b, k_{2\perp})
$$
$$
\times S_{ab\to cd}(\lambda_\perp) H_{ab\to cd}(P_\perp^2) \delta^{(2)}(\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{\lambda}_\perp - \vec{q}_\perp) \ . \tag{33}
$$

We can further simplify the polarized cross section by evaluating the expression

$$
\vec{e}_z \cdot \left(\vec{S}_\perp \times \hat{\vec{q}}_\perp\right) = \frac{|S_\perp|}{|q_\perp|} \vec{e}_z \cdot \left(\hat{\vec{S}}_\perp \times (\vec{P}_{1\perp} + \vec{P}_{2\perp})\right)
$$
$$
\approx \frac{|S_\perp|}{|q_\perp|} |P_\perp| (\sin\phi_1 + \sin\phi_2)
$$
$$
\approx |S_\perp| \left(\mathrm{Sgn}(\pi - \theta)\cos\phi_1 + \sin\phi_1 \frac{|q_\perp|}{2|P_\perp|}\right) \ , \tag{34}
$$

where ϕ_1 and ϕ_2 are the azimuthal angles of the two jets relative to the polarization vector $\vec{S}_\perp$, and $\theta \equiv \phi_2 - \phi_1$ the angle between the two jet transverse momenta. All these azimuthal

angles are defined in a frame that the polarized proton is moving the z direction. In the above derivation, we have used the approximations $|P_{1\perp}| \approx |P_{2\perp}| \approx |P_\perp|$ and $|q_\perp| \approx |P_\perp||\sin\theta|$, which are valid at small $q_\perp$ (θ is close to π). From the above result, we can see that there are two terms contributing to the spin asymmetry: one is with $\cos\phi_1$ and the other with $\sin\phi_1$. The first term has a Sign function associated, which gives a positive contribution when θ is smaller than π and a negative one otherwise.

In the above formulas, we did not include any gluon Sivers function contributions. The gluon Sivers function could dominate the asymmetry at central rapidities [33]. Another important issue is the relevant Sudakov suppressions for the asymmetries, which was found to be sizable for the di-jet correlation in the RHIC energy range [33]. In the following numerical studies, as an order of magnitude estimate for these asymmetries, we will neglect these effects, which however should be taken into account in future more detailed studies.

Following the same procedure that we used for the Sivers asymmetries in SIDIS and Drell-Yan processes, we can further simplify the polarized cross section by integrating out the transverse momentum, but keeping the azimuthal angle dependence explicit. The differential cross section then can be written as

$$\frac{2\pi d\sigma}{dy_1 dy_2 dP_\perp^2 d\phi_1} = \frac{d\sigma_{UU}}{dy_1 dy_2 dP_\perp^2} + \cos\phi_1 \frac{d\sigma_{TU}^{(1)}}{dy_1 dy_2 dP_\perp^2} + \sin\phi_1 \frac{d\sigma_{TU}^{(2)}}{dy_1 dy_2 dP_\perp^2} \ , \qquad (35)$$

where the unpolarized cross section has been given in Eq. (29), and the polarized ones read

$$\frac{d\sigma_{TU}^{(1)}}{dy_1 dy_2 dP_\perp^2} = \sum_{ab} x_a q_{Ta}^{(1/2)}(x_a) x_b f_b(x_b) \frac{d\hat{\sigma}}{d\hat{t}}(ab \to cd) \ ,$$

$$\frac{d\sigma_{TU}^{(2)}}{dy_1 dy_2 dP_\perp^2} = \frac{M}{|P_\perp|} \sum_{ab} x_a q_{Ta}^{(1)}(x_a) x_b f_b(x_b) \frac{d\hat{\sigma}}{d\hat{t}}(ab \to cd) \ , \qquad (36)$$

with the distribution $q_T^{(1)}$ defined as

$$q_T^{(1)}(x) = \int d^2 k_\perp \frac{k_\perp^2}{2M^2} q_T(x, k_\perp) \ . \qquad (37)$$

We note that the second term in the polarized cross section is power-suppressed by $M/P_\perp$. This suppression is due to the fact that we have integrated over all intrinsic transverse momentum. Clearly, this term is beyond the approximations we have made, and we cannot reliably predict it since there will be other sources of power-suppressed contributions, for example generated within the Qiu-Sterman mechanism [14]. Since it is anyway expected to be small, we will discard it in the following. Employing the same set of Sivers functions

that we used for our predictions for the Drell-Yan process above, we then find the results for $A_N = d\sigma_{TU}^{(1)}/d\sigma_{UU}$ shown in Fig. 13. We show the asymmetries as functions of the rapidity of jet "1", and of the jet transverse momentum $P_\perp$. One can see that the SSA for the di-jet correlation can become very large, in particular in the forward rapidity region. Asymmetries of this size should be relatively easily measurable in the future. We also note that the asymmetry has opposite sign compared to that for Drell-Yan dimuon production discussed earlier. The reason for this is that u-quark contributions dominate in Drell-Yan, thanks to their large electromagnetic charge, whereas for di-jets, d-quark contributions are not charge-suppressed and in fact dominate, keeping in mind that the analysis of the HERMES data appears to favor a large d quark Sivers function. The opposite signs of the Sivers up and down quark distributions we found in Eq. (19) then explains the opposite signs of the spin asymmetries for Drell-Yan and di-jets. We note that if the two jets are within the central rapidity region, our prediction for the asymmetry is much smaller. As mentioned above, the gluon Sivers function could dominate the asymmetry in this region [33].

We stress again that if factorization can be shown for the Sivers SSA in di-jet production, it is likely that the structure of the resulting expression may differ from the one we use. In particular, there will be calculable factors that represent the non-universality of the Sivers functions related to the process dependence of the gauge links, leading effectively to modified partonic hard-scattering functions [34], at variance with our use of the standard unpolarized ones. As a test, we have also used the modified partonic cross sections derived in [34]. We find relatively small changes in the results we obtain. Unfortunately, however, this is not really representative: the cross sections given in [34] are only for the quark-(anti)quark scattering channels, whereas the dominant contribution in our calculation mostly comes from qg scattering (with the gluon from the unpolarized proton). It remains to be seen to what extent eventually our predictions will change, once the process-dependence for the Sivers functions in di-jet correlations is completely understood.

We finally note that it would also be interesting – in particular for measurements with the PHENIX detector – to study correlations between hadrons in opposite jets. Such di-hadron correlations could serve as surrogates for the di-jet correlations we have discussed above. In this case there will, however, also be contributions from the Collins mechanism.
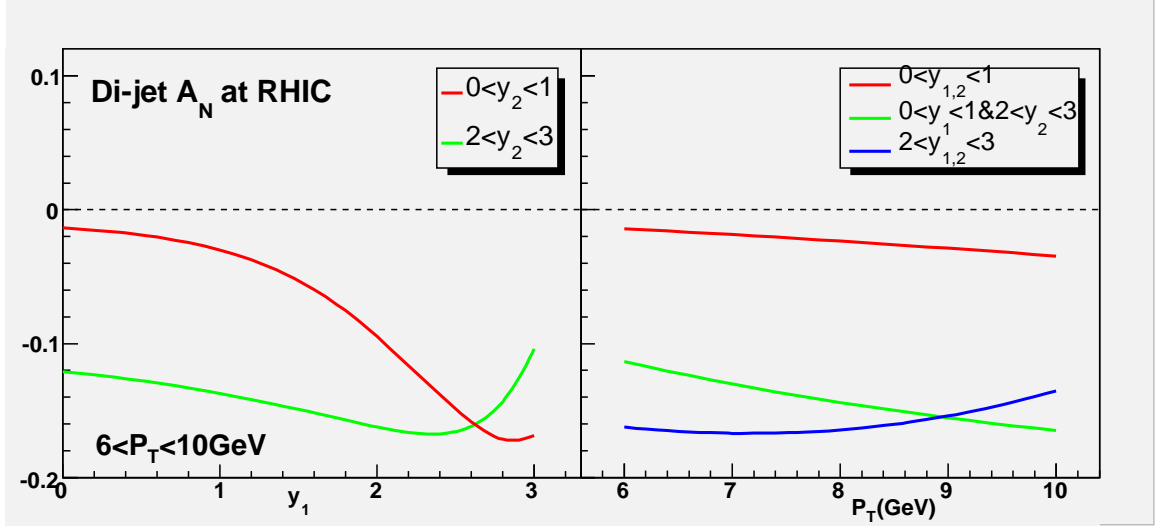
FIG. 13: *Sivers asymmetries for di-jet correlations at RHIC, as functions of rapidity y_1 and transverse momentum $P_\perp$.*

C.   Jet-Photon Correlations in $p^\uparrow p \to \mathrm{jet} + \gamma + X$

It is straightforward to extend the analysis of di-jet correlations discussed above to the case of jet-plus-photon correlations. We simply need to implement the cross sections for the appropriate Born-level partonic scatterings $q\bar{q} \to \gamma g$ and $qg \to \gamma q$ in Eq. (36). Although events with a photon suffer from smaller rates than two-jet events, they would offer additional information on the Sivers functions. It is also likely that proofs of factorization are more easily obtained here, since the reactions $q\bar{q} \to \gamma g$ and $qg \to \gamma q$ each have only a single color structure. Figure 14 shows results for the single-spin asymmetry for jet-photon correlations for the same kinematics as for the di-jet case in Fig. 13. Variables with the subscript "1" denote photon variables. Again, sizable asymmetries are seen, in particular at forward rapidities. The asymmetries are somewhat smaller than the ones we found for di-jets. This is a result of cancellations between our Sivers u and d functions, due to the larger weighting factor $4/9$ that the u-quark contributions now have for the prompt-photon case.

IV.   CONCLUSIONS

In this paper we have studied single-transverse spin asymmetries in semi-inclusive deep inelastic scattering and at hadron colliders. We have analyzed the Sivers and Collins con-
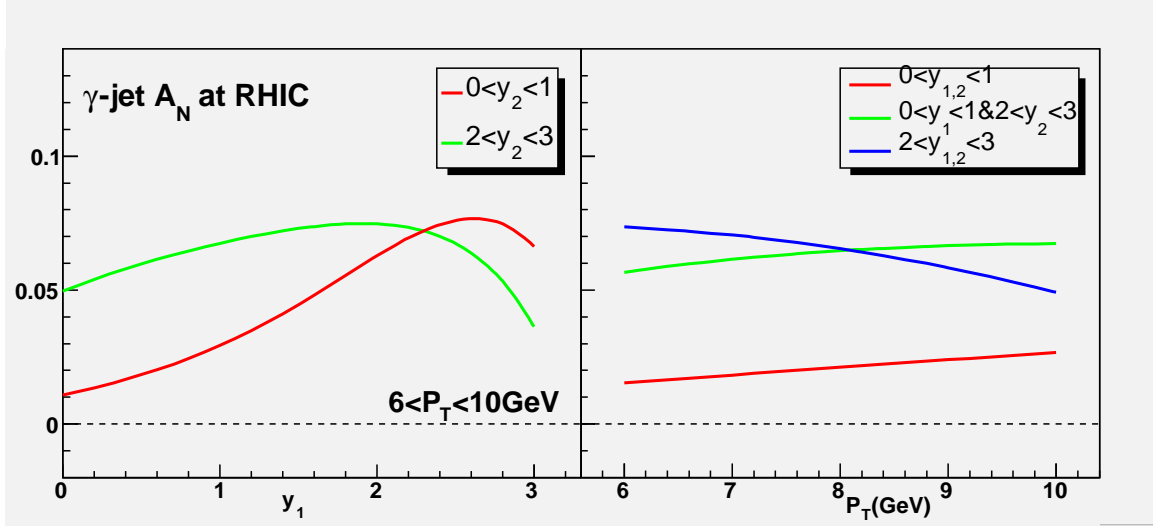
FIG. 14: *Sivers asymmetries for jet-photon correlations at RHIC, as functions of photon rapidity* y_1 *and transverse momentum* $P_\perp$.

tributions to the spin asymmetry in SIDIS, and fitted simple parameterizations of the corresponding functions to recent data from HERMES. These fits work well and also turn out to be consistent with COMPASS measurements of the asymmetries in DIS off a deuteron target. For the Sivers functions, we found dominance of the down-quark distribution over the up-quark one. The Sivers-d density in SIDIS turns out to be positive, while the u-quark distribution comes out negative. Concerning the Collins functions, we have found that current data do not yet pin down the relative size of "favored" and "unfavored" functions, which is also due to the fact that the transversity densities are not yet known. We have also given theoretical arguments that the "favored" and "unfavored" Collins functions could be of similar size, and of opposite sign.

We have then investigated Sivers-type single-spin asymmetries at hadron colliders, focusing on the Drell-Yan process and on di-jet and jet-photon correlations, all in circumstances where there is a small measured transverse momentum, but the process is overall characterized by a large scale. Using the Sivers functions obtained from the analysis of the HERMES data, we have made predictions for single-spin asymmetries for these processes. We find relatively large asymmetries, in particular at forward rapidities of the observed final state. Such asymmetries should be measurable with dedicated efforts at RHIC. Besides the additional valuable information they would give on the Sivers functions and therefore on the structure

of the nucleon, they would also provide a test of our theoretical understanding of "naively time-reversal-odd" phenomena in QCD. The crucial issues in this are the factorization of the corresponding cross sections, and the universality of the Sivers functions, on both of which further theoretical work is required.

Acknowledgments

Note Added: Upon completion of this paper, we noticed the preprint [55] where also the Sivers functions were fitted to the new HERMES and COMPASS data, and predictions for SSAs in the Drell-Yan process were made. As far as we can see, our results are in qualitative agreement with those of [55], keeping in mind that their sign convention for the Sivers function is opposite to ours and to that in the "Trento conventions".

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